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בקשה לפטנט
Application for Patent

C:33827

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אני, (שם המבקש, מענו -- ולגבי גוף מאוגד -- מקום התאגדותו)
I (Name and address of applicant, and, in case of body corporate-place of incorporation)

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רמות ע"י אוניברסיטת תל אביב
רשות אוניברסיטאית למחקר שימושי ופיתוח תעשייתי בע"מ
ת.ד. 39296
תל אביב 61392
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(באנגלית) (English) ELECTRO-OPTICAL TECHNIQUES FOR ACHIEVING VARIOUS TYPES OF SUPER RESOLVING CAPABILITIES

hereby apply for a patent to be granted to me in respect thereof
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מבקשת פטנט from Application	*לבקשה/לפטנט to Patent/Appl.	מספר/סימן Number/Mark	תאריך Date	מדינת האיגוד Convention Country
No. ______ .מס dated ______ מיום	No. ______ .מס dated ______ מיום			
*יפוי כח: כללי/מיוחד - רצוף בזה / עוד יוגש P.O.A.: general / individual - attached / to be filed later - filed in case ______ הוגש בענין				
המען למסירת הודעות ומסמכים בישראל Address for Service in Israel Sanford T. Colb & Co. P.O.B. 2273 Rehovot 76122				

חתימת המבקש Signature of Applicant	
For the Applicant, Sanford T. Colb & Co. C:33827	היום 30 בחודש March שנת 1999 This 30 of March of the year 1999

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שיטות אלקטרו-אופטיות לקבלת מגוון יכולות של סופר רזולוציה

ELECTRO-OPTICAL TECHNIQUES FOR ACHIEVING VARIOUS TYPES OF SUPER RESOLVING CAPABILITIES

RAMOT UNIVERSITY AUTHORITY FOR APPLIED
RESEARCH & INDUSTRIAL DEVELOPMENT LTD.

~~רמות- ע״י אוניברסיטת תל אביב~~

רשות אוניברסיטאית למחקר שימושי ופיתוח תעשייתי בע״מ

Inventors: David Mendlovic and Zeev Zalevsky

הממציאים: דוד מנדלוביק וזאב זלבסקי

C: 33827

Application for Patent

Title:

Electro-Optical Techniques for Achieving Various Types of Super Resolving Capabilities.

שיטות אלקטרו-אופטיות לקבלת מגוון יכולות של סופר רזולוציה.

Inventors:

1. David Mendlovic
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Assignment:

Ramot, Tel-Aviv University

Abstract:

The proposed principal aims to obtain geometrical and diffraction super resolving capability. The combination between the two types of techniques is most important since if only one type of improvement is obtained the resolution is to be restricted by the other type. The approach for obtaining the geometrical super resolution is based upon viewing the same spatial information several times while each time the image is viewed by a proper sub pixel shift. The image with the high spatial resolution is resolved by applying a Gabor transform like algorithm. The fine reconstruction is possible only due to special mask attached to the sensor plane. The mask may be prepared as a special optical element. The sub pixel viewing of the image may be obtained using the time multiplexing approach or the space multiplexing using a special optical element that replicates the imaged object with proper sub pixel shift among the replicas.

The diffraction super resolution ability may be obtained by various techniques. One of the most common is the one that involves two rotated gratings where the first one is attached to the viewed object and the second is attached to the sensor. Since in imaging systems, the object is located far away from the observer the attachment of the grating becomes a non-practical task. In scanning systems such as barcodes readers, the object-attached grating may be synthesized by illuminating the object by illumination having a spatial structure of a grating. The illumination is to be shifted with time in order to simulated the temporal movement of the grating.

1. Scientific background

The super resolution of imaging systems is an important subject for technological investigation and innovation [1-3].

In the literature, the studies of near or far field super resolution addressed issues intended to overcome diffraction phenomena [4-8]. The resolution of a system limited by diffraction is defined as the finest detail that can pass through the system without being distorted [9]. However, in many practical viewing systems, the resolution is limited by the detection sampling system, i.e. the resolution of the system sensor. This defines another type of super resolution, to be called "geometrical super resolution".

In this patent we will focus on this type of super resolution suggesting an algorithm that improves it. In addition we will adjust the diffraction super resolving techniques previously discussed in the literature [8], for the use of scanning systems. Regarding the geometrical super resolution, the approach for obtaining such a super resolution is to apply a time multiplexing (or a space multiplexing) procedure together with a sub pixel scans of the scene. Thereafter, electronic computation of the inverse Gabor transform [10] of the captured images will provide a single image with improved geometrical

resolution. Such an improvement is to be achieved only due to special masking of the sensing device. Regarding the diffraction super resolution, the adjustment is obtained by avoiding the attachment of a moving grating to the observed object. Instead, the object is to be illuminated by a grating pattern, which is changed with time in order to simulate the required time movement of the grating.

2. SW Adaptation Process

Refs. [11,12] propose a general tool for designing and analyzing optical systems that provide diffraction superresolution capability. A generalization of the space bandwidth product (SW) term is done there in order to map the distribution of the degrees of freedom within the phase space (in that particular example – a Wigner chart). Associating the SW to a function instead to a single number does it. The resolution merit is now the area, i.e the shape of the SW function, in the x, v_x plane. Using a distinction between the SW of a signal (SWI) and the SW of a system (of the ensemble average of the signals that the system can transfer perfectly, coined SWY) an adaptation algorithm that adapted the two shapes in the x, v_x plane was performed to achieve a complete transmission of information. This representation where only the area of the SW is important is relevant only for binary Wigner charts. The real number of degrees of freedom needed for fully expressing the signal is the volume of this SW function (proportional to the energy). Following this interpretation, the basic idea of any superresolution system is as follows: *Assuming that the optical system provides sufficient degrees of freedom for handling the expected input signal, but the degrees of freedom are not well distributed in the phase space to fit the system degrees of freedom distribution. Now, the superresolution action means an adaptation of SWI to SWY.*

Fig. 1 shows an example of such a 2-D SW – adaptation process. This example illustrates how the SWI is adapted to the SWY in terms of area adaptation. This process has been generalized [11] to except various degrees of freedom of the signal including time, wavelength, and polarization.

The 3-D (volume) adaptation process may actually be performed in two steps. First by performing the adaptation of the dynamic range and second by performing the 2-D (area) SW adaptation process previously mentioned. An important tool for performing the dynamic range adaptation operation may be a grating. Multiplying a signal by a phase grating, for instance, creates several replicas in the Wigner plane while each replica has lower intensity. This mean that a tradeoff is performed between the energy of each replica, which is decreased (lower energies need lower dynamic range), and the number of replicas (increasing the occupied area of the SW function). After the adaptation of the dynamic range, the 2-D SW adaptation algorithm will deal with the obtained area enlargements and will perform an adaptation between this area of the signal to the acceptance area of the system (by using other domain. as the time domain for instance, in the time multiplexing approach).

An example of geometrical superresolution in terms of SW adaptation is illustrated schematically by Fig. 2. This is the case where the spatial resolution of the viewed background is much finer than the spatial resolution to be viewed by the sensing device. Fig. 2a is the SW function of the signal (SWI) and Fig. 2b is the accepted SW of the system (SWY). In order to increase the viewed spatial resolution, SW adaptation process is applied. First, some of the dynamic range degrees of freedom are converted to spatial degrees of freedom using a grating for example. The SWI after this stage is shown in Fig. 2c. Then, based on time multiplexing (using the temporal degrees of freedom), each time slot, a part of the SWI shown in Fig. 2c is transferred. Fig. 3 demonstrates additional example where 3-D SW adaptation process was applied.

In this case [illegible]daptation was performed over the detector which is a part of system's characterizations. The adaptation was done by dividing each detector's pixel into three regions. This operation allowed the detector to sense spatial frequencies which are three times higher. but it decreased its dynamic range. This operation adapted the SWY to the SWI charts and allowed full transfer of signal's information.

3. Sub Pixel Methodology

Assume that the sensing device samples the line scene N times, each time with M pixels. Each sensing pixel integrates all of the light imaged upon it within the cycle. The detected energy of the k-th pixel (k=1,2,...,M) is thus:

$$y_k = \int_{-\infty}^{\infty} u(x) g(x-(k-1)\Delta x) dx \qquad (1)$$

where u(x) is the viewed object, Δx is the pixel size and g(x) is the spatial shape (i.e. apodization) of the pixel. In each adjacent sample, the image is shifted with respect to the device by a distance of $\Delta x/N$. Based, therefore, on the assumption that the sensing array is shifted with respect to the image (N-1) times, every time by sub pixel distance of $\Delta x/N$, the signal readout corresponding to the k pixel in the n th shift is:

$$y_k(n) = \int_{-\infty}^{\infty} u(x) g\left(x-(k-1)\Delta x - n\frac{\Delta x}{N}\right) dx \qquad (2)$$

The displacement of the CCD is equivalent to a movement of the scene u(x) along the opposite direction:

$$y_k(n) = \int_{-\infty}^{\infty} u\left(x + n\frac{\Delta x}{N}\right) g(x-(k-1)\Delta x) dx \qquad (3)$$

We now have N captures associated with each pixel. We will now sequence the $y_k(n)$ captures in an interlaced fashion, i.e. $y_m = y(m) = y([k-1]N+n)$. Assuming that the sensing array of the camera has M pixels, we now have

$$y_m = \int_{-\infty}^{\infty} u\left(x + (m-1)\frac{\Delta x}{N}\right) g(x) dx \qquad (4)$$

$m = 1,2,\ldots,MN$

Performing now a discrete time Fourier transform (DTFT) of the obtained y_m series having the length of MN terms yields:

$$Y(\Phi) = \sum_{m=1}^{MN} y_m \exp(-i2\pi(m-1)\Phi) \qquad (5)$$

where Φ is the frequency coordinate of the DTFT. We will now show that it is possible to reconstruct a discrete sampled version of the function u(x) with these MN samples. This will be done in spite of the fact that the sensor provides only M pixels per line and each pixel represents the convolution of u(x) with the pixel spatial modulation g(x). Let us assume that the spectrum (Fourier transform) of the continuous function u(x) is:

$$U(\nu) = \int_{-\infty}^{\infty} u(x) \exp(-i2\pi\nu x) dx \qquad (6)$$

A discrete time Fourier transform (DTFT) based on the MN samples is defined by:

$$U(\Phi) = \sum_{m=1}^{MN} u\left((m-1)\frac{\Delta x}{N}\right) \exp(-i2\pi(m-1)\Phi) \qquad (7)$$

Note that Φ (the DTFT spectral coordinate) and ν (the Fourier transform spectral coordinate) are related via the sampling rate

$$\Phi = \frac{\Delta x}{N}\nu \qquad (8)$$

A well-known relation between the Fourier transform and the DTFT is:

$$U(\Phi) = \sum_{l} U\left(\nu - l\frac{N}{\Delta x}\right) \qquad (9)$$

when l gets values so that the spectral width of $U(\nu)$ is fully scanned.

Substituting Eq. 4 into Eq. 5 and changing the order of summations (the integral and the summation), one gets:

$$\sum_{m=1}^{MN} u\left(x+(m-1)\frac{\Delta x}{N}\right)\exp(-i2\pi(m-1)\Phi)$$
$$= U(\Phi)\exp\left(i2\pi x\frac{N}{\Delta x}\Phi\right) \qquad (10)$$
$$= U(\Phi)\exp(i2\pi x\nu)$$

Now, returning back to Eq. 5, one obtains:

$$Y_m(\Phi) = \int_{-\infty}^{\infty} g(x)\exp(i2\pi\nu x)dx\,U(\Phi)$$
$$= G(-\nu)\sum_{l} U\left(\nu - l\frac{N}{\Delta x}\right) \qquad (11)$$

where $G(\nu)$ is the Fourier transform of $g(x)$.

As N increases, l runs along smaller range and the aliasing between the different replica of U in Eq. 11 (different l values) is decreased. The result is that the spectral band of $Y(\Phi)$ is increased (a larger amount of spatial frequencies is viewed in the captured image).

In order to resolve $u(x)$, one needs to divide $Y(\Phi)$ (the DTFT of Y) by $G(-\nu)$ and thereafter to return to the spatial domain by performing and inverse DTFT:

$$U(\Phi) = {Y(\Phi)}\big/{G(-\nu)} \qquad (12)$$

Indeed, special care should be devoted to frequencies were $G(-\nu)$ gets zero or becomes close to zero. This can be done using standard deconvolution approaches taking into account that commonly the pixel shape $g(x)$ is symmetric and thus $G(-\nu)$ is real. In addition since the pixel is more sensitive to light at its central area and less at the its outer regions, the sensitivity function $g(x)$ is similar to a Gaussian function whose Fourier transform has only non zero values.

This operation of image capturing described by Eq. 1 is actually equivalent to the performance of the discrete Gabor transform over u with the window function g. Note that performing a DTFT followed by a division by G and then performing an inverse DTFT is equivalent to the performance of the following discrete convolution:

$$y_m = \sum_{p=1}^{MN} g\left(p\frac{\Delta x}{N}\right)u\left((m-p)\frac{\Delta x}{N}\right). \qquad (13)$$

In order to implement the DTFT by computer one should perform the discrete Fourier transform (DFT) operation which is equivalent (in proper conditions) to performing a sampling of the DTFT. The DFT may be calculated using fast algorithms as the fast Fourier transform (FFT).

Since in the space domain we wish to obtain MN samples, we need to sample the DTFT spectrum in distances that are:

$$\Delta\Phi = \frac{1}{MN}. \qquad (14)$$

This is since the DTFT coordinate Φ period range is bounded between -½ and ½ due to the periodicity of the DTFT spectrum. Padding the spatial series with zeros (enlargement of its length) decreases $\Delta\Phi$. Since according to Eq. 12 the algorithm includes division by $G(-\nu)$ (which may have zeros due to its spatial limited sizes) we will choose $\Delta\Phi$ in a way so that the sampling points $\Delta\Phi k$ (k is an integer) will not coincide with the zeros of $G(-\nu)$.

4. Masking Operation

According to Eq. 12 the algorithm requires division by $G(-\nu)$, which may have zeros due to its spatial limited sizes, an optical device is needed in order to remove the zero regions beyond the maximal frequency to be reconstructed after the sub pixel procedure. A fine transmission grating with period of Δx could be attached to the sensing device panel. Such a transmission grating will decrease the effective size of each pixel so that its Fourier transform $G(-\nu)$ will have wider band and the zero regions will be removed to higher frequencies. The apodized pixel (determined by the transmission pattern of the grating) should be designed in an iterative manner so that the bandwidth of its Fourier transform $G(-\nu)$ will be wider than the highest frequency aimed to be reconstructed via the sub-pixel procedure. The price to be paid by this solution is the

decrease of the energy sensed by the pixels.

Returning now to the computation estimation requirement of the proposed algorithm, we find that it is in the order of MN length Fourier transform for each dimension. Indeed, a fast Fourier transform routine can be used, but still, the amount of calculations is high.

One can reduce dramatically the computation requirements if some apriori information on the object shape is known. For example, if the task is to localize a light spot with a given width. Then, by ignoring diffraction distortions, the spot will activate a finite response only in a well known number of neighbored pixels. This can reduce the number of calculations involved in the Fourier/inverse Fourier transform calculations.

The masking shape may be determined, for example, as exhibited by Fig. 4. Fig. 4a illustrates the input object u(x). Its Fourier transform is seen in Fig. 4b. Fig 4c presents the pixel's shape g(x) and Fig. 4d is its Fourier transform. In Fig. 4e one may see the sampled u(x) before the sub pixel super resolution algorithm and in Fig 4f its Fourier transform. Fig. 4g presents u(x) sampled with a resolution eight times higher than the one in 4e. Fig. 4h is its Fourier transform. Fig. 4k presents the shape of the improved masking and Fig. 4l shows the expansion of the spectrum of g(x) due to the improved masking. The resulting spectrum does not contain zeros.

Fig. 5 illustrates the operation of the sub pixel algorithm. In Fig. 5a a barcode is exhibited. Due to the restricted spatial resolving ability of the sensing device, the captured image is the one seen in Fig. 5b. After creating the masking modification, using 8 vertical replications and eventually applying the sub pixel algorithm, the obtained result is seen in Fig. 5c. A comparison between Fig. 5a and 5c exhibits the successful spatial reconstruction.

5. Mask Positioning

In infra red imaging systems (FLIRs) it is quit complicated to places a special mask attached to the sensor due to cooling problems. In most of the telescopic imaging systems a mid imaging plane exists. In this plane an imaging of the output scene is obtained. This plane is also imaged over the sensor with a magnification factor usually of one. Instead of placing the required mask attached to the sensor one may place it in the mid imaging plane where the mask may be cooled without any special problems. Since the diffraction restrictions are:

$$\delta x = 2.44\lambda F_\#,$$

where $F_\#$ is the F number of the lens, the fine spatial frequencies required from the mask will be able to pass to the sensor and to allow to operate the geometrical super resolution algorithm.

6. Computer Simulations

In order to demonstrate the abilities of the suggested technique several computer simulations were performed for 2-D objects. The object presented in Fig. 6a which has spatial resolution of 256*256 pixels was captured by a CCD camera having a spatial resolution of only 32*32 pixels and 8 quantization bits of dynamic range. Fig. 6b illustrates the low resolution image seen by the CCD before applying the super resolution algorithm. The mask attached to each pixel of the CCD was [0 1 0 1 1 0 0 0] where 1 indicates full transmission and 0 indicates zero transmission.

The reconstructed image after applying the algorithm is presented in Fig. 6c. One may see that all of the spatial details primly existing within the input image were fully reconstructed.

In Fig. 7 we wish to examine the sensitivity of the algorithm to the number of quantization bits of the CCD camera. Fig. 7a illustrates the low resolution image

captured a CCD having 32*32 pixels and only 1 quantization bit. Fig. 7b is the reconstructed image. One may see that it is not identical to the input but due to the masked time multiplexing sub pixeling algorithm, the obtained result is much closer to the original input not only by its spatial resolution but also by the gray level range obtained in each pixel. Fig. 7c is the image captures by the same CCD but with 2 quantization bits per each pixel. Fig. 7d is the reconstructed image. One may see that it is almost identical to the original one. Fig. 7e presents the obtained reconstruction when a 4 bit CCD camera was used. Here as well the image was fully reconstructed. From the above simulations we may conclude not only that using the masked time multiplexing sub pixeling algorithm the spatial resolution is fully reconstructed, but also the obtained result is not sensitive to the number of quantization bits of the CCD. Even for a CCD having 2 quantization bits, the original image is almost completely reconstructed.

7. Ways to Implement the Geometrical Super Resolution

The sub pixel shifting of the captured image may be achieved by time multiplexing or by space multiplexing. The time multiplexing approach requires an active micro scan in which the sensor scans the field of view in a sub pixel shifts. This scan may be achieved by rotated mirrors, which reflect the scene to the sensor. Another way is to use a passive time multiplexing approach, which can be applied if the sensor is placed on a vibrating platform. In this case, the parameters of the vibration are to be estimated by special vibration sensors or by algorithmic means. Then, the sampling of the view is to be synchronized by a special timer such that the sampling will occur exactly when the vibrated sensor is located at the required sub pixel shift.

In the space multiplexing approach a special grating like optical element (which can be a diffractive or a refractive optical element) may be attached to the aperture of the objective lens. The element causes to the desired replications, of the imaged scene, over the sensor's plane (a Fourier transform of a grating is a set of delta functions).

8. Diffraction Super Resolution

Ref. [8] speaks about a diffraction super resolution technique, which is obtained using two special rotated gratings placed attached to the object and to the sensor. The movement of the grating attached to the object plane encodes its spatial information by a Doppler like effect and allows transmitting it through the restricted aperture of the imaging lens. The decoding of the information is obtained using the second grating attached to he sensor plane. This configuration may be very useful for microscopic applications but it fails to be practical in imaging systems. In applications dealing with scanning systems, the special moving grating, which is to be attached to the object, may be synthesized by the illumination source. A special diffractive optical element may be attached to the illumination source such that in the object plane a grating pattern is to appear and to scan the object. By phase modulating the source a movement of this pattern is to be obtained on the object plane (linear phase modulations are expressed in the far field approximation as shifts). This effect may be achieved also by illuminating the object with two coherent beams. Due to the coherence of the beams, interference fringes will appear. By phase modulating one of the beams the fringes will be shifted appropriately.

9. **What is claimed is:**

1. The algorithm which includes spatial resolution improvement using space multiplexing and containing:
   a) An unconventional (diffractive, reflective or combined) optical element placed instead of the imaging lens to create replicas of the imaged object, if a space multiplexing approach is used and a special sampling-timing device if the sensor is placed on a vibrating platform and a time multiplexing approach is used.
   b) A masking that modifies the pixel's shape to avoid zeros in the Fourier transform of g(x).
   c) A sub pixel algorithm, e.g. a Gabor transform based algorithm, for achieving the resolution improvements.
2. The method of claim 1, wherein said that a different sub pixel algorithm is used instead of the Gabor transform (e.g. Wavelet transform or a Mellin transform if a non-equalized shifts are created between the different replicas).
3. The method of claim 1 wherein said that the sub pixel scan is done using the time multiplexing approach with micro scan.
4. The method of claim 1 wherein said that the sub pixel scan is done using the time multiplexing approach by extracting the parameters of a vibrating platform.

5. The method of claim 1 wherein said that for the space multiplexing sub pixel scan, the design of the optical element which creates the desired replicas a Gerschberg-Saxton algorithm or multi facet implementation may be applied.
6. The method of claim 1, wherein said that some other type of multiplexing is obtained in order to achieve the resolution improvement. For instance wavelength multiplexing.
7. The method of claim 1, wherein said that former or following the spatial super resolving algorithm a noise removal technique is applied. The noise suppression technique may be a Wiener filter, morphological correlator, triple correlator or any other known procedure.
8. The method of claim 1, wherein said that the mask is placed in mid imaging plane.
9. The method can be applied also for 2-D objects while any of the known multiplexing approaches may be applied over the 2-D replications of the object (for instance time, space or wavelength multiplexing). If again a spatial multiplexing is used the object may be replicated as a 2-D object or the replication may be applied on the object as line by line.
10. A device that obtains a diffraction super resolution using the technique described in Ref. [8] and a special illumination which replaces the grating attached to the observation scene. The movement synthesize of this virtual grating is to be achieved using a phase modulation of the illuminating source or by phase modulating one out of two interfering beams creating desired interference fringes.

For the Applicant,

Sanford T. Colb & Co.
C:33827

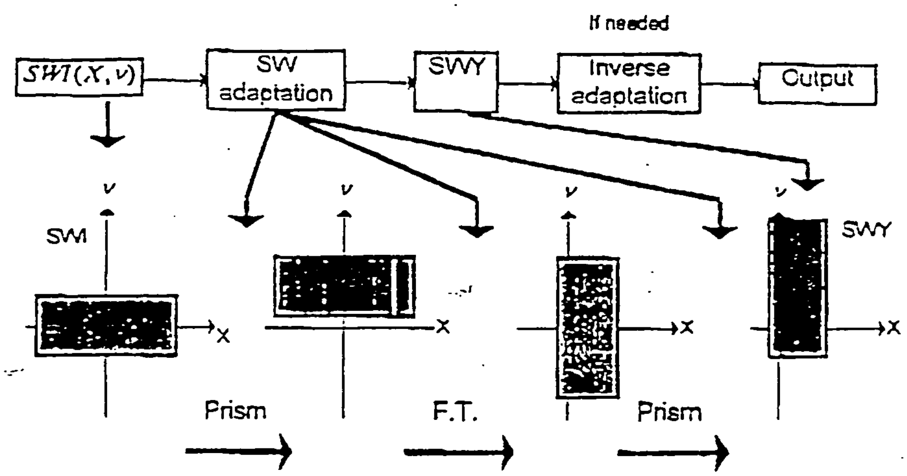


Fig. 1 An example of the SW-adaptation process.

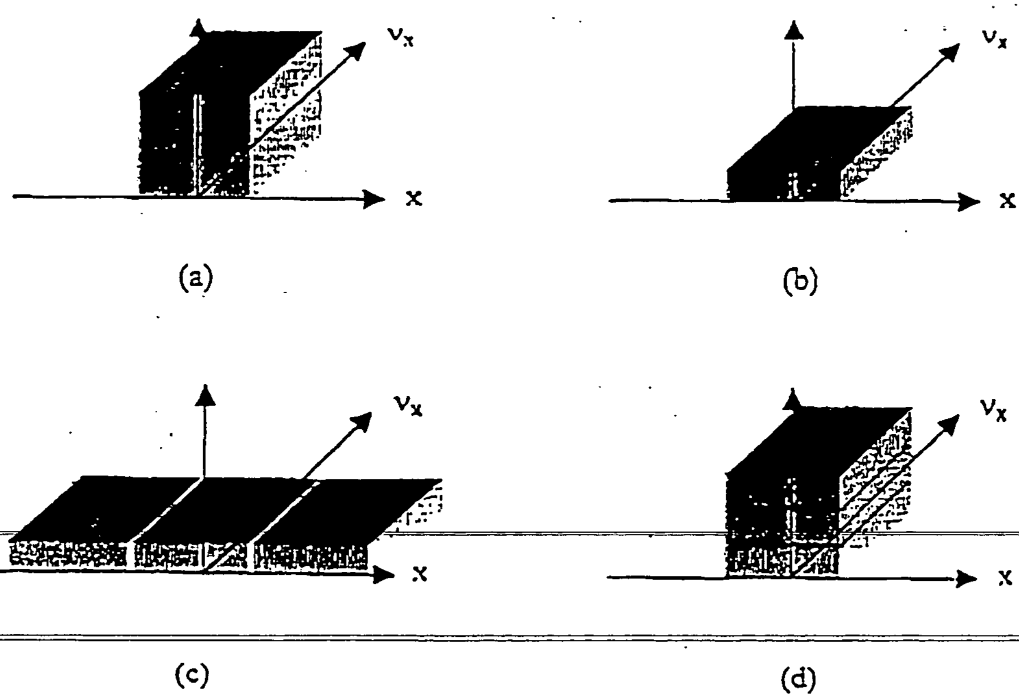


Fig. 2 The 3-D SW adaptation process illustrating dynamic range trade off operation. (a) SWI, (b) SWY, (c) SWI after grating and (d) SWI after time multiplexing.

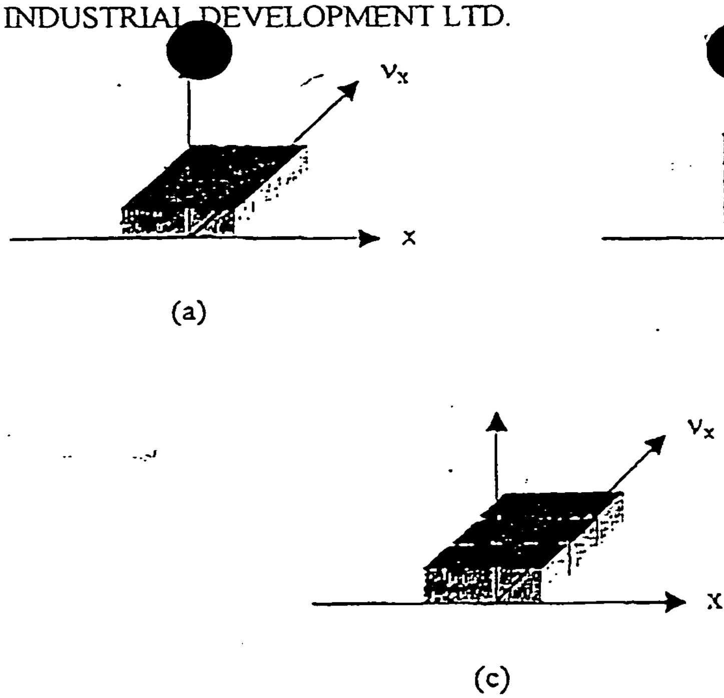
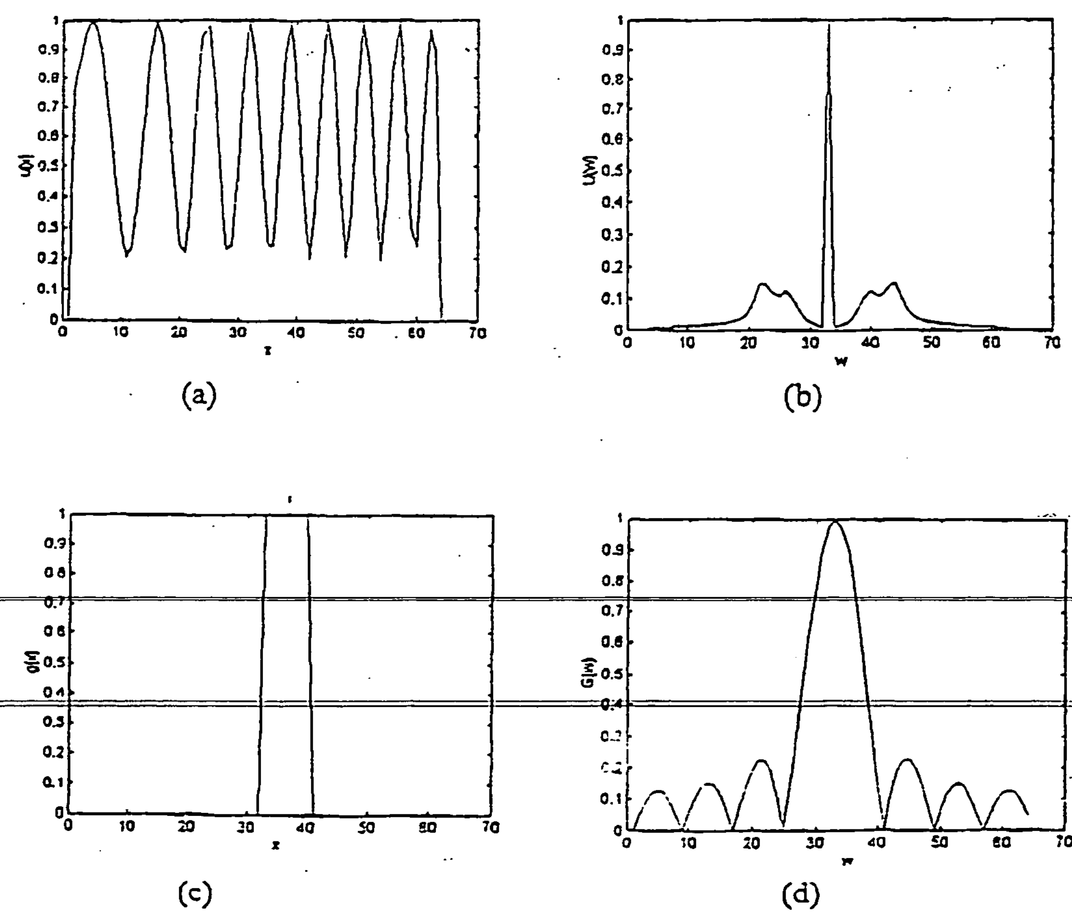
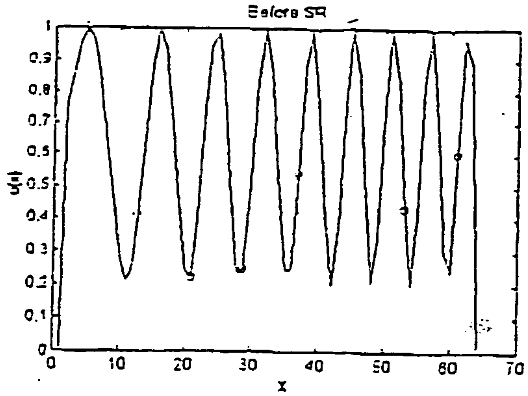


Fig. 3 The 3-D SW adaptation process illustrating sub-pixeling operation. (a) SWI, (b) SWY, (c) SWY after dividing each pixel to 3 regions.





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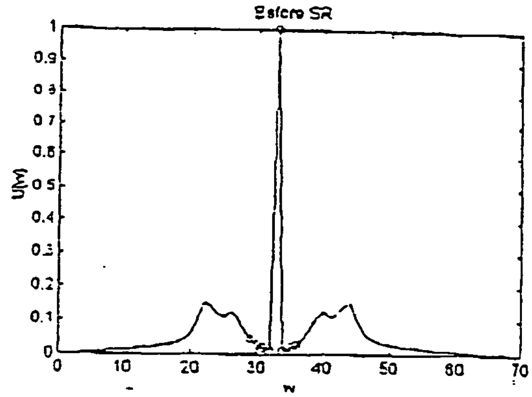
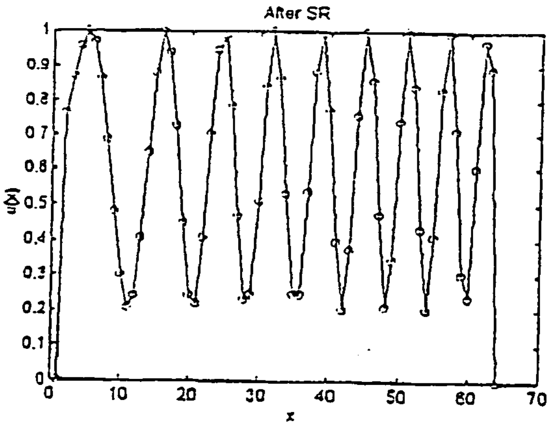


(f)



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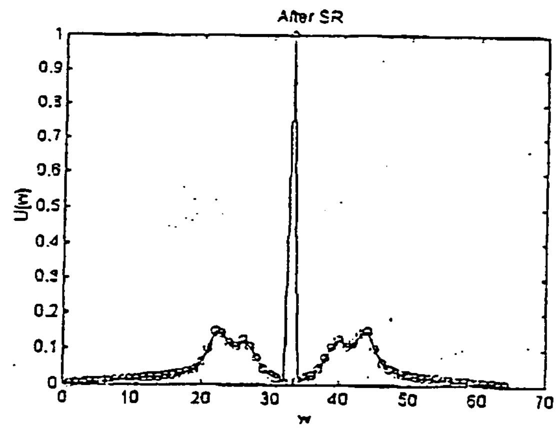
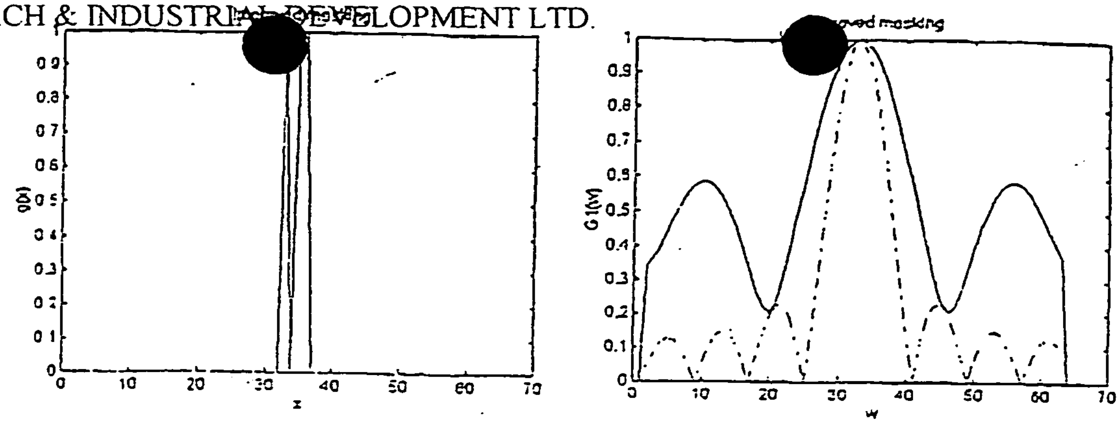
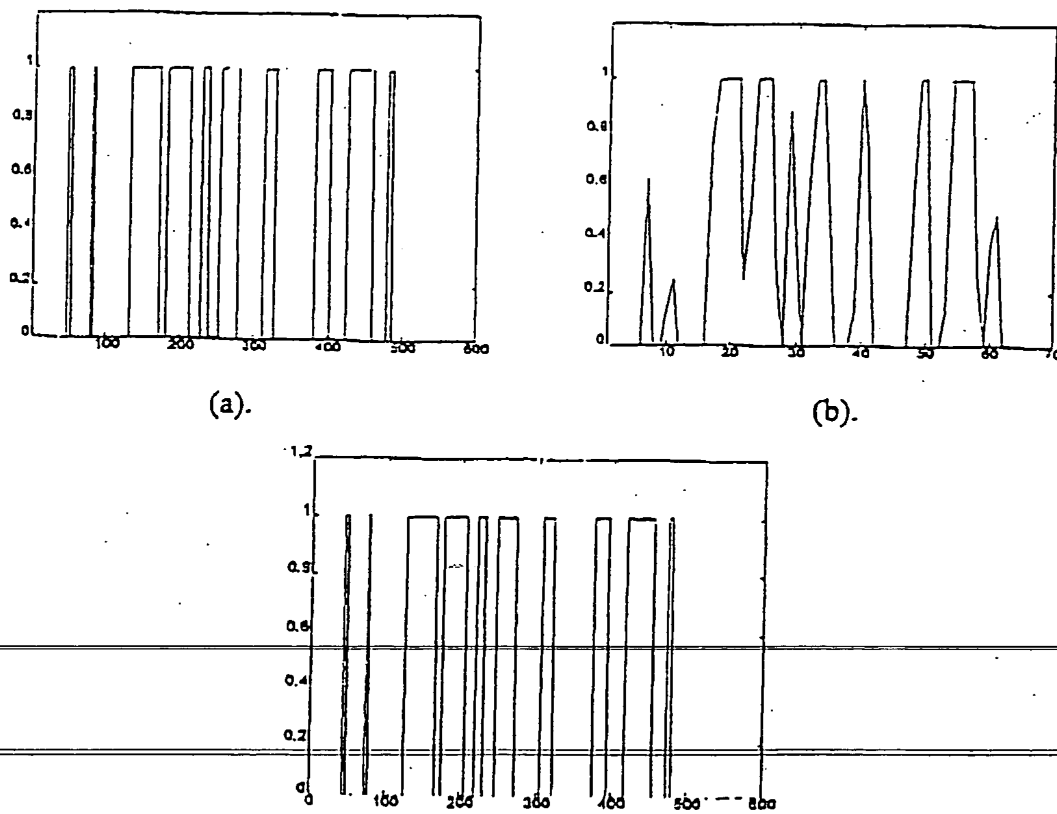


(h)



(k) (l)

Fig. 4 Avoiding division by zeros algorithm. (a) u(x), (b) Fourier transform of u(x) (c) g(x) (d) Fourier transform of g(x) (e) sampled u(x) before sub-pixel super resolution(f) its Fourier transform (g) u(x) sampled with a resolution eight times higher than the one in (e) (h) its Fourier transform (k) the shape of the improved masking (l) the expansion of the spectrum of g(x) due to the improved masking.

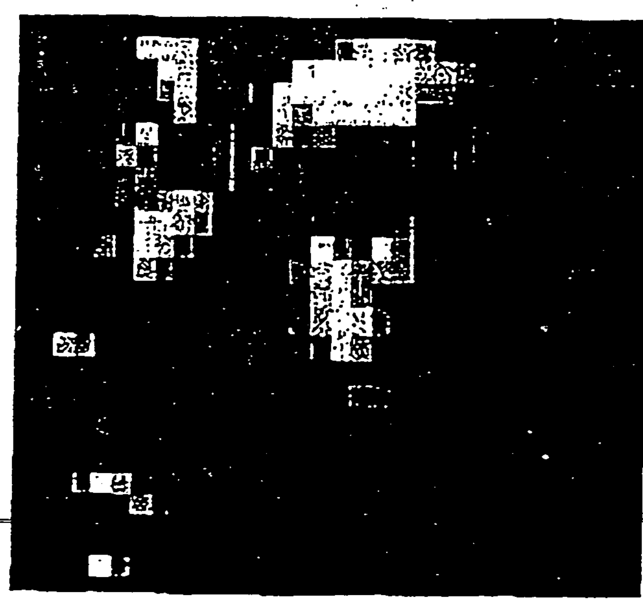


(a). (b).

(c).

Fig. 5 The sub pixel super resolving algorithm. (a) the original object (b) the low resolution captured object. (c) the reconstruction via the geometrical super resolving technique.

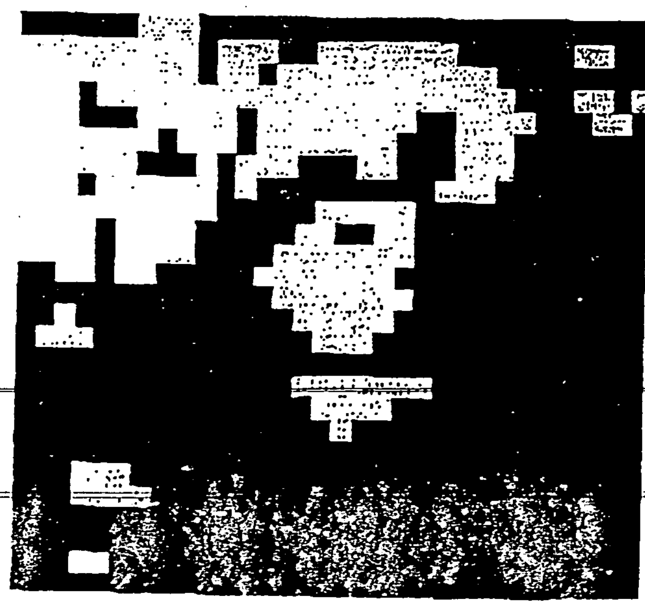
(a)



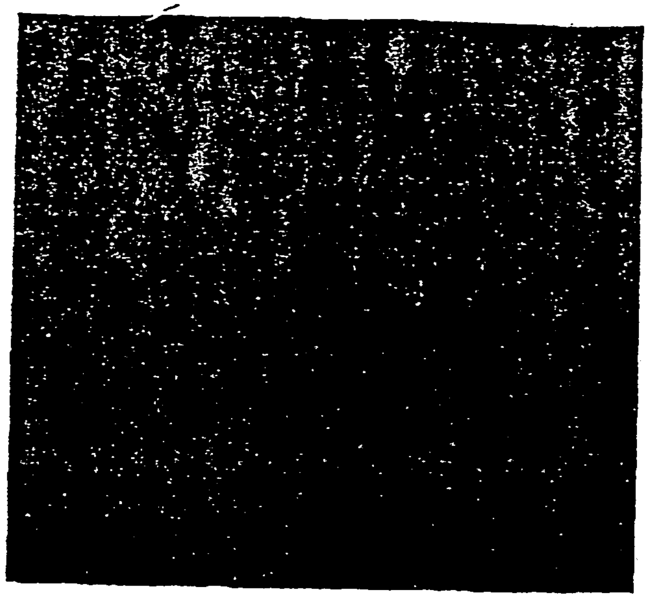
(b)

(c)

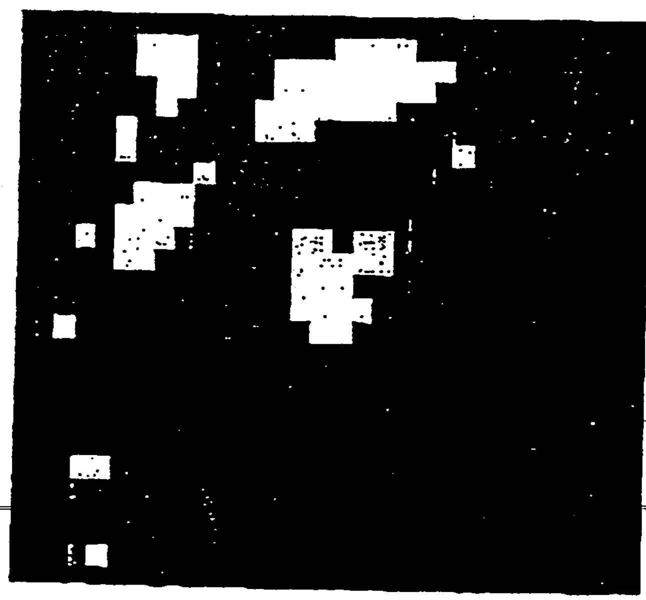
Fig. 6 Computer simulations of the suggested technique. (a). Original image, (b). Image after resolution decreasing (c). Reconstructed image.



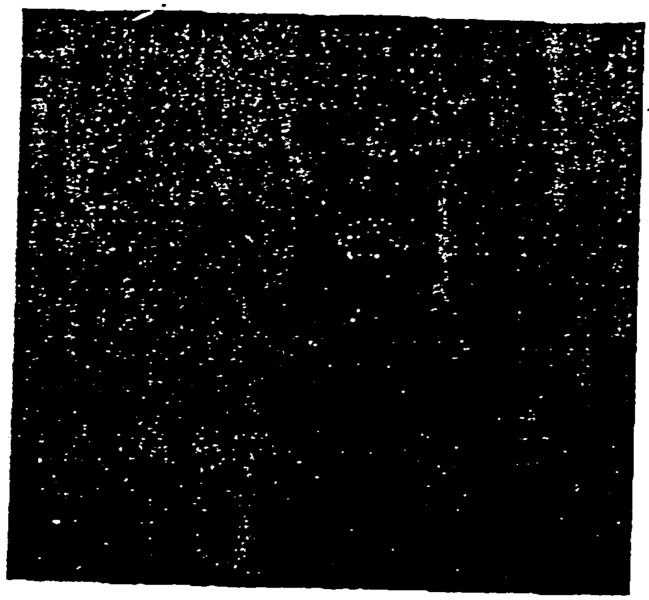
(a)



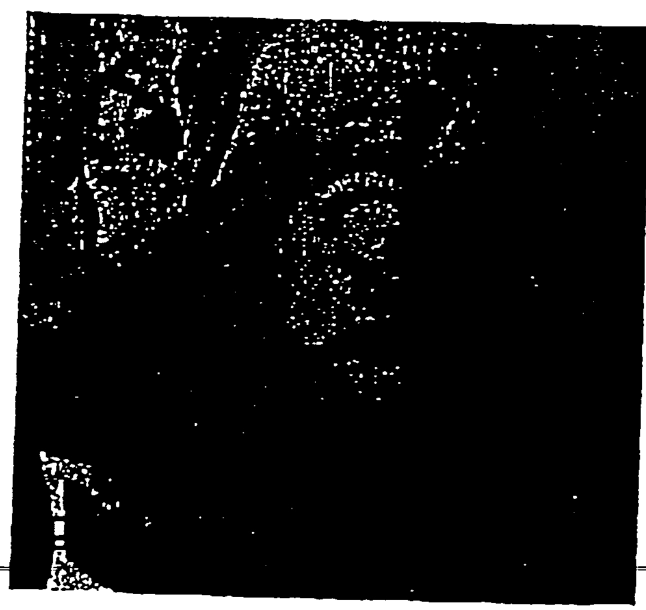
(b)



(c)



(d)



(e)

Fig. 7: Computer simulations that examine the sensitivity of the suggested technique to the number of quantization bits of the camera. (a). Low resolution image captured by a CCD with 1 quantization bit. (b). The reconstructed image. (c). Low resolution image captured by a CCD with 2 quantization bits. (d). The obtained reconstruction. (e). The reconstruction of an imaged that was captured by a CCD with 4 quantization bits.